TEACHING MATHEMATICS DISTINCTIVELY

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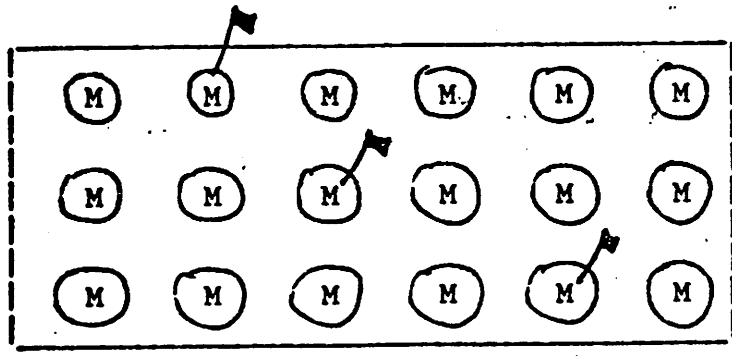
Let me first make three disclaimers. First, I recognize that there is something rather pompous and vain about someone who speaks on the topic of teaching mathematics distinctively especially to an audience such as this. Lurking in the vicinity is the tacit assumption that such a person is an expert in this area and will serve as a model for the suggestions he makes. Such is certainly not the case with me. Please regard what I say as something to which I aspire and not something I practice completely. Second, by way of disclaimer, I recognize that distinguishing oneself may be thought of as setting oneself apart for special attention and praise. Again, this is not the situation I have in mind. I mean to distinguish oneself as a servant of God and fellow men when I talk about "distinguished teaching."

Finally, what I say is not to be viewed as prescriptive for you. Each of us must develop our own ways of serving God using our individual talents and gifts and also taking into account the situation in which He has placed us.

Now to return to the subject at hand: How to teach mathematics distinctively. Of course, we need a definition of "distinctive mathematics teaching." Do you agree that one's teaching career is an odyssey in exploring the meaning of this concept? In my own case there has been a gradual change in my thinking and practicing of what teaching mathematics distinctively means. Let me trace some of these changes by describing two mathematical models that I have been using.

Model 1: The Narrow Road.

In this model Professor X, who has ambitions to teach mathematics distinctively, views herself as one of many mathematics teachers and tries very hard to distinguish herself from other teachers of mathematics by the sheer excellence of her classroom performance. There are at least two ways to proceed. One can polish and hone the skills and techniques of effective teaching by carefully preparing lectures spiced with good examples and counter-examples. Once can model precise language and one can give careful proofs for the main theorem. One can develop the skill to use the blackboard wisely and well. One can exhibit excellent style in problem-solving; one can skillfully use overhead projectors and colored chalk. Put succinctly, one can distinguish oneself from the crowd of other mathematics teachers by being an excellent practitioner of the trade of teaching mathematics. So, let us refer to the geometric diagram. We can put a little flag on those teachers of mathematics who distinguish themselves in this way.



Now what I say later on in my talk should not be interpreted as denigrating attempts that all of us make to be distinctive in this sense. It will take a life-time of sustained effort to perfect the arts and skills in the trade of teaching mathematics and we do well to spend time and effort in perfecting them.

Yet there is another dimension to all of this which we mathematics teachers who are Christians are well aware of. This holds especially true for those who teach in Christian colleges, but is an aspiration of all of us who attend this conference. In this setting we seek to be distinctive by integrating our faith and learning by pointing out to our students the relations we perceive between the Christian faith and the discipline we teach. For mathematics professors this often consists of a critique of the discipline of mathematics in historical context pointing out the strengths and deficiencies of the methodology of mathematics and also delving into the philosophical problems associated with it. In my case, (mirroring my Calvinistic tradition) this integration culminates in an attempt to muster a list of propositions, the first of which are clearly theological and biblical in content, passing through proportions which are philosophical in character (ontology, epistemology, etc.), and ending with propositions which are mathematical. The glue which holds all of these statements is pure logic. Thus, one demonstrates that there is a vital relationship between Christianity--one's religious commitments, and the theoretical work in the discipline of mathematics. (Let us put a second flag on those teachers that are able to distinguish themselves in this way.)

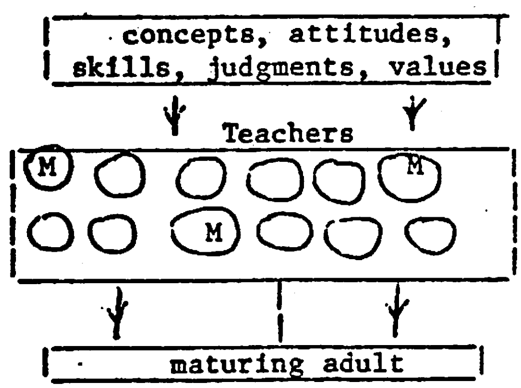
To summarize, the basic idea is this. The Christian scholar-teacher who is an active participant in the on-going research of the discipline of mathematics and who has thought deeply about the relationship between his religious convictions and the discipline of mathematics and who communicates these effectively to his students in the classroom is a distinctive teacher of mathematics.

Again, please do not infer from what I say that I am down-playing all attempts that are being made to make the analytical connections between the Christian faith and the cultural efforts of man as evidenced in the various disciplines. We have all engaged in this exercise and I, for one, have found it to be personally worthwhile and stimulating. These Wheaton conferences stand as a record of our attempts to do some of this analysis communally. I believe that some real progress has been made.

Yet, I have found in my own teaching that there is something unsatisfactory in my own attempts to integrate my faith and learning on a purely analytical level. For one, the burden is almost entirely on me in that the material is handed down to the student for his scrutiny. Even my "best" attempts are received with a "ho-hum" attitude (in much the same way that the standard textbook material is received!). Sad to say, such "distinctive" teaching may not get to the core of the student and may not effect her will, her emotions. In short, I consider the two descriptions of our attempts to be distinctive as too narrow and somewhat deficient for the very reason that they do not sufficiently take the student into account. Let me now present what I take to be a more satisfactory view which encompasses the more narrow view and which builds on it.

Model 2: The Wider View

This model takes into account the fuller picture that the educational process involves at least three components. There is the area of what we teach our students (concepts, skills, values, judgments, etc.), there are the people and methods we use to communicate to students, and there is the student himself/herself--the maturing young adult. I believe that we can enhance our effectiveness as teachers by taking each of these components into account. A more faithful description of what is taking place is that we are only one class of teachers who are communicating with students are that we have opportunity to contribute uniquely to the total development of the students we teach.



Such a holistic viewpoint requires that we get a more complete description of the students we teach. Recently I found in an Appendix to our President's Report to the Board of Trustees some material produced by our Student Affairs Division which was very helpful to me.* It contained a description of current psychosocial and development theories concerning the maturing young adult. Let me summarize two of these theories.

The first is a psychosocial theory developed by Arthur Chickering using ideas originally proposed by E. H. Erikson. Chickering gives seven major developmental vectors in the development of college students! These vectors are the following:

(1) Achieving competence: this includes the development of intellectual and social abilities as well as physical and manual skills.

(2) Managing emotions: becoming aware of personal feelings and recognizing that they provide information relevant to contemplated behavior or to decisions about future plans.

*A Self-Study by the Student Affairs Division of Calvin College. Report III, Appendix M, President's Report to the Board of Trustees (April 1981).

(3) Becoming autonomous: becoming emotionally independent (freedom from continual need for reassurance and approval) and instrumental independence.

(4) Establishing self-identity and becoming aware of the importance of accepting interdependence.

(5) Freeing interpersonal relationships: expressing greater trust, independence, and individuality in relationships with others; developing tolerance.

(6) Clarifying purposes and priorities.

(7) Developing integrity: selecting a set of beliefs that have some internal consistency and provide a guide for behavior.

The report also contains a description of the cognitive-developmental theory as proposed by William Perry. Perry describes the intellectual and ethical development of students as occurring in a sequence of stages. They are as follows:

(1) Dualism: the dualistic learner assumes that all information can be classified as right or wrong (in discrete, absolute categories). To learn is to find the right answer. (At first blush it appears that the study of mathematics contributes to this viewpoint.)

(2) Multiplicity: the student recognizes multiple perspectives but, being unable to judge adequately, draws the conclusion that one answer is as good as another.

(3) Relativism: students recognize that knowledge is contextual and relative. She begins to see the big picture and begins to realize that some choices have to be made.

(4) Commitment in relativism: students make an active affirmation in a pluralistic world.

When I read this list I am struck by the complexity of the total developmental process of the students we teach. A natural question arises. In which ways do we mathematics teachers and the discipline we teach contribute to this total development of the maturing young adult?

The report I cited also includes a more theological Biblical description of the student which I am including.

(1) Each student bears the image of God, possessing both dignity and potential, distinguished from all the other creatures by the rational, emotional, volitional, and spiritual qualities.

(2) Each student, though having a natural tendency to hate God and neighbor, can find forgiveness and renewal in Jesus Christ and is called to live in that newness of life.

(3) each student is unique and deserves to be understood and assisted in terms of that individuality.

(4) Each student has the potential to develop, thereby achieving higher stages of cognitive, psychological, and spiritual maturity.

(5) Each student has the major responsibility for his/her development and must make the choices and bear the responsibility for the choices which can promote or hinder instruction.

(6) Each student develops within, is responsible to, and must be served by a community.

In terms of the foregoing, I am now prepared to give a definition of distinctive teaching of mathematics.

A Definition of distinctive teaching

To teach mathematics distinctively is to creatively use the unique aspects of the discipline of mathematics to join with fellow teachers of the various subjects in the furthering of the full development of the college students we teach.

According to this definition, we must enlarge and broaden the experiences we provide in our classrooms so that we touch more aspects of the student's development. Such holistic teaching will affect the strategies we employ in the class. I suggest that it may mean at least the following:

(1) There should be more emphasis upon the response of the student in the classroom setting. To teach distinctively in the classroom means to get your students to be active participants in the on-going activities of the classroom. This means that students join in the formulation of problems, in the framing of careful definitions, in the making of conjectures, in the construction of proofs and in deciding where the mathematics can be applied.

(2) There should be more emphasis upon the individual effort of the student in learning the material covered and in using a variety of assisting materials that are available.

(3) There should be more emphasis in the classroom and in the assignments made upon the historical-philosophical context of the subject matter covered.

(4) The student should be offered a variety of ways to respond to the material to be learned. This means that not only should the students be given daily problem assignments but also a succession of carefully chosen articles to be read and analyzed to supplement the material covered in class. Such assignments will require the student to write summaries of these articles but also, more importantly, to react to them in written form. I find that there are many articles that are available which will require some careful analysis and discernment to discover controversial assertions about mathematics and which require sensitivity to issues in order to make a good response. These articles may be written from a Christian perspective, but often are more valuable when they are not and present views which are antithetical to a Christian position. It is indeed gratifying to get papers written by students that contain perceptive analyses of issues presented in articles they are assigned to read. I am also gratified to report that students enjoy reading them.

(5) Furthermore, I believe that it is important to offer a variety of outside projects that students may do which apply the mathematics covered in class. Such projects may include the use of the computer or apply the techniques covered to an area outside of mathematics. In any case, the student should be notified early in the course via the course syllabus, perhaps, that such a project is required. Of course, it will be necessary to make suggestions for such projects and also to consult with students on their progress throughout the semester.

(6) There should be more emphasis placed upon the mathematics community of the college which a student joins when he or she becomes a mathematics major. I am a firm believer in the necessity of a mathematics club or colloquim where students can hear talks and presentations of various students and eventually make their own contribution.

(7) As we develop rapport with our students and as we listen to the variety of responses they give, we will be given opportunity to tell the students of our own personal commitments. I have found that the formal lecture is less effective in this regard than the informal exposing of ourselves and our opinions as a result of conversations with students. If they are faced with questions, if they are made to realize that mathematics carries with it a multitude of puzzling and interesting questions and if we can get the student to begin to formulate their own answers to them, then we ourselves will participate more fully, in a natural way, in the expressing of our deepest feelings to our students.

In closing, let me illustrate how this may be done by describing the various reading assignments and projects I offered students in the Mathematical Statistics class I taught last semester.

<u>Selected Readings for a Course in Mathematical Statistics</u>

The first thing to do is to think of some important questions you want to raise for the consideration and reaction of the students. They might be:

(1) What does a professional statistician do? What are some of the fundamental questions that statisticians answer? Are there some disagreements about what methodology to adopt?

(2) Am I qualified to become a statistician? Should I become one? Are there various discrete areas in statistics to choose from?

Now some theological-philosophical questions.

(3) How does God providentially care for His world? Are His dealings with the natural world summarized by mathematical laws? Does God deal with the creation order through stochastic, probabilistic processes? What is a religious account of determinism and randomness?

(4) How does one explain the fact that man's mathematics (seemingly a product of his mind) has applications and predictive value in the "real" world around us?

Continuing, there are questions about how statistics can be used and misused.

(5) Are there some ethical norms that a Christian should be aware of when he does statistical sampling and experimentation? Should any statistical study which is well-posed mathematically actually be carried out?

(6) Are there rules or norms that one should apply in the reporting of statistical findings? How would you report the findings of a statistical study you carried out in a public media?

Lastly,

(7) How is statistics used in such diverse areas as economics, biology, psychology, etc.?

Here are some readings that can be used.

1. H. O. Hartley: Statistics as a Science and as a Profession, Journal of the American Statistical Association, March 1980, Vol. 75, No. 369 (Presidential Address).

2. Nelson Goodman: Mathematics as an Objective Science, Mathematics Monthly, Vol. 86, No. 7, August-September 1979, pp. 540-551.

3. R. W. Hamming: The Unreasonable Effectiveness of Mathematics, Mathematics Monthly, Vol. 87, No. 2, February 1980, pp. 81-90.

4. Bradley Efron, Controversies in the Foundations of Statistics, Mathematical Monthly, Vol. 85, No. 4, April 1978, pp. 231-246.

5. Morris Kline: Mathematics in Western Culture, Oxford University Press, 1964. Chapters XXII, XXIII, XXIV.

6. Huff, How to Lie with Statistics.

7. David S. Moore: Statistics: Concepts and Controversies, W. H. Freeman & Company, 1979, Chapters 1 and 2.

8. D. B. Owen, On the History of Statistics and Probability, Marcel Dekker, Inc., 1976, Chapter 7.

The Emergence of Mathematical Statistics written by Jerzy Neyman.

Students were also assigned a course project in which they posed a statistical question, they obtained data for analysis, they performed the relevant tests, they drew statistical conclusions, and finally they wrote a report summarizing their findings.

Appendix

Some interesting and provacative articles that can be used in the mathematics classroom.

1. Erst Snapper — The Three Crises in Mathematics: Logicism, Intrutionism and Formalism. Mathematics Magazine Vol. 52, No. 4, September 1979, pp. 207-216.

2. Thomas Tymozcko — Computers, Proofs, and Mathematicians: A Philosophical Investigation of the Four-Color Proof. Mathematics Magazine, Vol. 53, No. 3, May 1980, pp. 131-138.

3. *R. W. Hamming — The Unreasonable Effectiveness of Mathematics. Math Monthly, Vol. 87, No. 2, Feb. 1980, pp. 81-90.

(Man, so far as we know, has always wondered about himself, the world around and what life is all about. We have many myths fromthe past that tell how and why God, or the gods, made man, and the universe. These I call theological explanations. They have one principal characteristic in common, there is little point in asking why things are the way they are, since we are given mainly a description of the creation as the gods chose to do it.)

4. P. R. Holmes — The Heart of Mathematics. Math Monthly, Vol. 87, No. 7, August-September 1980, pp. 519-524.

5. E. R. Swart — The Philosophical Implications of the Four-Color Problem. Mathematics Monthly, Vol. 85, No. 9, 1980, pp. 697-707.

6. Dorothy L. Beinstein — The Role of Applications in Pure Mathematics, Vol. 86, No. 4, April 1979, pp. 245-253.

7. *Nicholas D. Goodman — Mathematics as an Objective Science, Math Monthly, Vol. 86, No. 7, August-September 1979, pp. 540-551.

Surfacism: Principle of Objectivity. Anything which is practically real should be taken as objectively real.

8. Philip J. Davis — Are There Coincidences in Mathematics? Math Monthly, Vol. 88, No. 5, May 1981, pp. 311-320.

9. Bradley Efron — Controversies in the Foundations of Statistics, Math Monthly, Vol. 85, No. 4, April 1978, pp. 231-246.

10. Felix E. Browder — The Relevance of Mathematics, Math Monthly, Vol. 83, No. 4, April 1976, pp. 249-254.

(Mathematics I, II, III, IV) transcendental ideal of math knowledge.

11. P. J. Davis — Fidelity in Mathematical Discourse: Is One and One Really Two?, Math Monthly, Vol. 79, No. 3, March 1972, pp. 252-263.

12. N. Bourbaki — The Architecture of Mathematics, Math Monthly, Vol. 57 (1950), pp. 221-232.

13. H. B. Griffiths — 1871: Our State of Mathematical Ignorance, Math Monthly, Vol. 78, No. 10, Dec. 1971, pp. 1067-1085.

14. John Myhill — What is a Real Number? Math Monthly, Vol. 79, No. 7, Aug-Sept. 1972, pp. 748-754.

15. R. L. Wilder — History in the Mathematics Curriculum: Its Status, Quality, and Function, Math Monthly, Vol. 79, No. 5, 1972, May, pp. 479-495.

16. Judith V. Grabiner — Is Mathematical Truth Time Dependent? Math Monthly, Vol. 81, No. 4, April 1974, pp. 354-365.

17. J. D. Monk — On the Foundations of Set Theory, Vol. 77, No. 7, Aug-Sept. 1970, pp. 703-711.

18. Vern S. Poythress — A Biblical View of Mathematics (Foundations of Christian Scholarship, pp. 159-188.

19. Chandler Davis — Where Did Twentieth-Century Mathematics Go Wrong? Paper for Science & Technology Meeting, Oct. 1980, University of Toronto.

(Platonist metaphysics and rejection of applications are two manifestions of a single trend)...

(The aridity of our courses, their remoteness from student's human concerns--together, of course, with their difficulty--make them especially forbidding, hence especially good as selectors of students with superior capacity for self-displine...)

(Indeed there is a clear affinity between the "modern" mathematics of the category, the scheme, and the topos, the "modern" music of the row and the cluster, and the "modern" painting and poetry of multiple uses...)

(Indeed, I think time has run out. If we set an intellectual isolation spinning our fantasies, our share of the available scientific solaries will steadily diminish, and our prestige still more rapidly.)